

## Optimal Monetary Policy at the Zero Lower Bound on Nominal Interest Rates in a Cost Channel Economy

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### **Abstract**

*It has been widely debated what a central bank should do to stimulate the economy when the nominal interest rate is at the zero lower bound (ZLB). The optimal monetary policy literature suggests that monetary policy inertia, i.e., committing to continue a zero interest regime even after the ZLB is not binding, is a way to get the economy out of recession. In this paper, I examine whether this result holds when monetary policy has not only the conventional demand-side effect but also a supply-side effect on the economy. To accomplish this objective, I incorporate the cost channel of monetary policy into an otherwise standard new Keynesian model and evaluate the optimal monetary policy at the ZLB. The study reveals several important insights in the conduct of the optimal monetary policy in a cost channel economy at the ZLB. Importantly, the discretionary policy requires central banks to keep interest rates at the ZLB for longer in a cost channel economy compared to no-cost channel economies. Further, cost channel economies introduce a policy trade-off between inflation and output gap. In contrast, under commitment policy, the simulation exercise shows that the central bank is able to terminate the zero interest rate regime earlier in a cost channel economy than otherwise. It is also revealed that the cost channel generates substantially high welfare losses, under both discretionary and commitment policies. Accordingly, abstracting the cost channel in these types of models can lead to under estimation of welfare losses.*

**Key Words:** *Optimal Monetary Policy, Zero Rates on Nominal Interest Rates, Cost Channel of Monetary Policy, New Keynesian Model, Liquidity Trap*

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## 1. Introduction

The zero lower bound on nominal interest rates (ZLB) is no longer just a theoretical interest. Nominal interest rates were at zero lower bound in the recent past in many countries across the globe.<sup>1</sup> It has been widely debated what a central bank should do to stimulate the economy when the aggregate demand is weak, even when the nominal interest rate is at zero level.<sup>2</sup> Optimal monetary policy literature suggests that monetary policy inertia, i.e. committing to continue a zero interest regime even after the ZLB is not binding, is a way to get the economy out of recession. In this paper, I examine whether this result holds when monetary policy has not only the conventional demand-side effect but also a supply-side effect on the economy.

In optimal monetary policy literature, there are two main policies which attempt to stabilise the economy in terms of inflation and output following a shock to the economy, namely, discretionary policy and commitment policy. Under discretion, the central bank takes the current state of the economy and private sector expectations as given. Under this policy, the central bank optimises in each period; therefore, any promises given by the bank are not credible. On the other hand, under commitment policy, the central bank chooses a path for current and future inflation as well as output and commits to that. Therefore, under commitment, if the central bank is credible, it can adjust private sector expectations [see Walsh (2010, pp. 357-364)]. Nobel laureates Finn E. Kydland and Edward C. Prescott, in their seminal paper "Rules Rather than Discretion: The Inconsistency of Optimal Plans" [henceforth; Kydland and Prescott (1977)], showed how an announcement of commitment to a low inflation regime by monetary authorities might create lower private sector inflationary expectations. They argued that if this monetary policy is then changed and interest rates are reduced to give a short-term lift to employment, credibility of policy makers will be lost and conditions may worsen.

However, literature on optimal monetary policy at the ZLB thus far has abstracted an important characteristic of the economy, i.e. the cost channel of monetary policy.<sup>3</sup> The cost channel is said to be present in an economy if the changes in nominal interest rates affect the

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<sup>1</sup> Some central banks have pushed short-term nominal interest rates below the zero lower bound. In the present study, I consider short-term nominal interest rates to be constrained by the zero lower bound.

<sup>2</sup> During the past decade, central banks around the world, including the Federal Reserve Bank of the USA had to resort to unconventional monetary policies due to ZLB constraints. Two main such unconventional policies that have been considered are forward guidance and balance sheet policies. However, in this paper, my focus is only on conventional monetary policy with the interest rate instrument.

<sup>3</sup> Chattopadhyay and Ghosh (2016) conducted a study on optimal monetary policy at the ZLB in a cost channel economy with varying degree of interest rate pass-through. Their methodology is similar to the methodology used in this paper and they report results similar to mine. Both papers were written in the same time period, but independently.

supply-side of the economy. It has been found by many recent studies<sup>4</sup> that the cost channel is an important channel of monetary policy in the USA and other developed countries. Ravenna and Walsh (2006), utilising a new Keynesian forward looking model, theoretically showed that the cost channel affects optimal monetary policy in important ways.

Presence of the cost channel could affect the policy outcomes of new Keynesian studies which examine zero lower bound policies. In general, the cost channel makes changes directly to the current inflation as well as the current output gap due to changes in nominal interest rates, other things being equal. In addition, when the cost channel is active in the model, it accelerates future inflationary expectations if the monetary authority commits to a tight monetary policy. This again raises current inflation. Consequently, the presence of the cost channel may affect the optimal monetary policy at the ZLB.

The main objective of this paper is to examine central bank policy options at the ZLB in a cost channel economy. Specifically, this study inquires when a central bank should exit the zero nominal interest rate regime. In this regard, I consider both discretionary and commitment policies, although the study mainly focuses on commitment policies. I consider a variation of the standard new Keynesian model to accomplish the above objective. To carry out simulations, I calibrate the model to the economy of the USA.

The main findings are as follows: a) the discretionary policy requires central banks to keep interest rates at the zero lower bound for longer in a cost channel economy compared to no-cost channel economies. b) Under commitment policy, the simulation exercise shows that the central bank is able to terminate the zero interest rate regime earlier in cost channel economies than otherwise. c) The cost channel generates substantially high welfare losses, under both discretionary and commitment policies.

The rest of the study is structured as follows. In section 2, I review the relevant literature on the optimal monetary policy at the ZLB and the optimal monetary policy with the cost channel. Section 3 describes the model, steady states and optimal dynamic paths. Model simulations and results are given in section 4. Section 5 concludes the study.

## 2. Literature Survey

In this section, I review the relevant literature under two main sections. Firstly, literature about optimal monetary policy at the ZLB is reviewed. Secondly, optimal monetary policy literature in a cost channel economy is reviewed. The main focus in the literature review is new Keynesian models as they are the most relevant to the present study.

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<sup>4</sup> For example, see Barth and Ramey (2001), Christiano et al. (2005), Kim and Lastrapes (2007), Henzel et al. (2009), Ravenna and Walsh (2006) and Chowdhury et al. (2006).

## 2.1 Optimal Monetary Policy at the ZLB

The evolution of the new Keynesian framework where management of private sector expectations is incorporated explicitly into economic modelling has improved the analysis of the optimal monetary policy at the ZLB. The importance of explicitly considering non-linearities in analysing the behaviour of the new Keynesian model at the ZLB has been shown by many scholars. Among them, Fernández-Villaverde et al. (2012) showed how the decision rules and the equilibrium dynamics of the model are substantially affected by the non-linear features at the ZLB.

Although there are few policy options within the traditional interest rate rule to get out of the ZLB constraint, the most accepted solution is the commitment to a policy. Krugman (1998) puts it as follows: "monetary policy will in fact be effective if the central bank can credibly promise to be irresponsible, to seek a higher future price level". He argues that under these conditions the liquidity trap boils down to a credibility problem. Private agents expect that any monetary expansion carried out by the central bank at the ZLB would be reverted immediately once the economy has recovered. Such expectations may not stimulate the economy in the recession. As a solution, Krugman suggests that the central bank should commit to a policy of high future inflation over an extended horizon.

Following Krugman's work, many proved in more complex dynamic models that the commitment to a policy plan which is facilitated by forward guidance is one way of getting out of the slump. Eggertsson and Woodford (2003) studied optimal commitment policy with ZLB in an inter-temporal model in which the natural rate of interest is allowed to take two different values. The natural rate of interest was assumed to become negative unexpectedly in the beginning and then move to a positive level with certain probability in every period. They explored how the existence of the zero lower bound affects the optimal conduct of monetary policy with regard to both inflation and output. Eggertsson and Woodford recommended a form of price-level targeting rule that should bring about the constrained optimal equilibrium if the central bank is credible. Jung, Teranishi and Watanabe (2005; henceforth JTW) considered a similar set up to Eggertsson and Woodford (2003) with perfect foresight, however, they considered an exogenous AR (1) process to the natural rate of interest. Both Eggertsson and Woodford (2003) and JTW found that at the ZLB, under commitment, the central bank should continue zero nominal interest rates even after the natural rate of interest returns back to the positive level. Doing so, the central bank can stimulate the economy by generating higher inflationary expectations. Extending this work, Hasui et al. (2016) considered the optimal commitment policy in an economy with inflation persistence. They argued that inflation persistence changes the central bank's objective from achieving target inflation rate to inflation smoothing. Therefore, agents expect an accommodative monetary policy, in turn, increasing inflationary expectations. This produces an acceleration in inflation and allows the central bank to terminate the ZLB policy earlier compared to an economy without inflation persistence.

The above line of research assumes that the central bank is fully credible, to the extent that private agents believe the commitments. Bodenstein et al. (2012) relaxed the assumption of the fully credible central bank. In a new Keynesian set up, they found that at the ZLB, the central bank faces a severe time-inconsistency problem. Initially, a promise to keep the nominal interest rate low for an extended period raises inflationary expectations. Further, it lowers current and future real interest rates, and thus stimulates current output. However, once the economy has emerged from the slump, it is not optimal to keep interest low any longer. Accordingly, they found that if a central bank's announced promises are not credible, then the economy goes through a deeper recession than otherwise.

All the studies specified above are based on the central bank optimising social welfare. There is another line of research which studies the performance of simple monetary policy rules at the ZLB. Here, the monetary authority commits to a particular type of rule such as the Taylor rule [Taylor (1993)]. Studies such as Fuhrer and Madigan (1997), Eggerston and Woodford (2003), Wolman (2005), Coenen et al. (2004), and Nakov (2008) examine this problem. These studies, in general, show that if the target inflation rate is closer to zero, simple policy rules such as the Taylor rule, can generate significant welfare losses. However, Eggerston and Woodford (2003) and Wolman (2005) showed that the policy rules formulated in terms of a price level target can considerably reduce these welfare losses. In contrast, Hasui et al. (2016) showed that the performance of price-level targets in an economy with inflation persistence is substantially low.

## 2.2 Optimal Monetary Policy with the Cost Channel

Ravenna and Walsh (2006) were the first to show that the presence of the cost channel alters the optimal monetary policy problem in important ways. They showed that the interest rate changes carried out to stabilise the output gap lead to inflation fluctuations when a cost channel is present. As a consequence, the output gap and inflation fluctuate in response to productivity and demand disturbances, even when the central bank is setting policy optimally. They assumed that a cost channel is present in the economy when firms' marginal cost depends directly on the nominal interest rate. Following Ravenna and Walsh (2006), others analysed the optimal monetary policy with the cost channel from different perspectives and found that the cost channel is important when analysing the optimal monetary policy. Chattopadhyay and Ghosh (2016), written independent of this paper, consider optimal monetary policy in a cost channel economy. They report similar results to this paper under both optimal discretionary and optimal commitment policies using a new Keynesian model at the ZLB. In addition to the two optimal policies, they consider a policy called the 'T-only' policy. Under the T-only policy, the central bank chooses and announces the optimal exit time of a zero interest rate regime and promises to exercise the discretionary policy following the exit. Chattopadhyay and Ghosh show that this policy closely replicates the commitment policy both under presence and absence of the cost channel.

Fiore and Tristani (2013) studied optimal monetary policy in a model of the credit channel with the cost channel of monetary policy. Using a second-order approximation of the welfare function, they showed that welfare is directly affected not just by the volatility of inflation and the output gap, as in the standard case where there are no financial frictions, but also by the volatility of the nominal interest rate and credit spreads. Credit spreads affect optimal monetary policy through the cost channel. Higher credit spreads make borrowing costly for firms by increasing marginal cost of production. Overall, the authors concluded that the monetary authorities ought to pay attention to financial market friction.

Tillmann (2009) studied the optimal monetary policy with an uncertain cost channel. He concluded that the larger the degree of uncertainty about the cost channel, the smaller the interest rate response to inflation. He incorporated uncertainty of the cost channel into the model since the effectiveness of the cost channel varies significantly over time and across countries. Therefore, the monetary authority may not be certain about the effectiveness of the true role of the cost channel at a given time. The framework of his study is new Keynesian, which has a policymaker who plays a zero-sum game against an evil agent who sets the parameters such that the welfare loss is maximised. In the model, an uncertain policy maker should overestimate the quantitative importance of the cost channel when setting interest rates. In this sense, the policymaker is less aggressive than under certainty.

Studies show that optimal monetary policy in the presence of the cost channel leads to an increased indeterminacy region. Surico (2008) studied the conditions that guarantee equilibrium determinacy in a standard sticky price new Keynesian model augmented with a cost channel. Surico showed that a central bank that assigns a positive weight to the output gap in the reaction function makes the economy more prone to multiple equilibria compared to the standard case. His results are robust to forward-looking, current, and backward-looking policy rules. Surico suggested that, when the cost channel is empirically important, trying to limit cyclical swings in real activity may result in undesired volatility of inflation and output.

The next section presents the model, derives steady states and analyses the optimal dynamic path following a negative shock to the economy.

### **3. The Model**

I consider a new Keynesian forward looking inter-temporal model to study the cost channel economy at the ZLB. This model is more suitable for the present analysis as it incorporates private sector expectations explicitly into the model. The model is based on JTW and Ravenna and Walsh (2006). I extend these authors' models to incorporate both the cost channel and the ZLB. The basic model is standard; however, a brief exposition is presented here to self-contain the analysis. The exposition is based on Ravenna and Walsh (2006); however, their model has been simplified by ignoring the government and taste shocks. Following them, I

assume the cost of labour must be financed at the beginning of the period. However, their assumption that the full labour cost has to be financed externally at the beginning of the year has been relaxed.

The model economy consists of three main sectors, namely, households, production and monetary authority. Financial intermediaries are also part of the economy, where firms borrow money to finance their wage bill. These players interact with each other in assets, goods and labour markets.

### 3.1 Households

There is a large number of identical infinitely-lived households in the economy. The preferences of a representative household are defined over a composite good  $C_t$  and time devoted to employment  $N_t$ . Households maximise the expected present discounted value of utility:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{C_{t+i}^{1+\eta}}{1+\eta} \right],$$

where  $\beta \in (0, 1)$  is a subjective rate of discount,  $\sigma > 0$  is the coefficient of relative risk aversion and  $\eta > 0$  is elasticity of labour supply. The composite consumption good consists of differentiated goods produced by monopolistically competitive final goods producers. There is a continuum of such producers of measure 1.  $C_t$  is defined as follows:

$$C_t = \left[ \int_0^1 C_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

where  $c_{jt}$  is the consumption of the good produced by firm  $j$  and  $\theta (> 1)$  is the elasticity of substitution between varieties. The price elasticity of demand for the individual goods is determined by  $\theta$ . As  $\theta$  increases, the different goods become closer substitutes. According to this specification, consumer demand and the aggregate price index are given by,

$$C_{jt} = C_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} C_t \quad \text{and} \quad P_t = \left[ \int_0^1 P_{jt}^{1-\theta} dj \right]^{1/(1-\theta)},$$

respectively. The price of the final good of firm  $j$  at time  $t$  is  $P_{jt}$ .

Households receive their labour income at the beginning of the period at the nominal wage rate of  $W_t$ . They enter the period  $t$  with cash holdings of  $M_t$  and make deposits  $D_t$  at the

financial intermediary. Accordingly, household's consumption expenditures are restricted by the following cash-in-advance constraint:

$$P_t C_t \leq M_t + W_t N_t - D_t$$

and budget constraint:

$$M_{t+1} + D_t + P_t C_t \leq M_t + W_t N_t + R_t D_t + \Pi_t$$

where  $\Pi_t$  is the profit income received from owning financial intermediaries and  $R_t$  is the gross nominal interest rate. It is also assumed that households are subject to a solvency constraint that prevents them from engaging in Ponzi-type schemes.

By maximising household utility subject to budget constraint, the following first order conditions (FOCs) are obtained:

$$C_t^{-\sigma} = \beta E_t \left( \frac{R_t P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}, \quad (1)$$

$$\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t}, \quad (2)$$

$$P_t C_t = M_t + W_t N_t + D_t. \quad (3)$$

The next section describes the production sector of the economy.

### 3.2 Production Sector

Firms in this model use no capital in the production process. They have to pay wages at the beginning of the period, before realising sales proceeds. The production technology is given by  $y_{jt} = A_t N_{jt}$ , where  $y_{jt}$  is total demand for good  $j$  in period  $t$ ,  $N_{jt}$  is employment by firm  $j$  in period  $t$  and  $A_t$  is an exogenous aggregate productivity factor. The staggered price setting of Calvo (1983) is used assuming each firm resets its price in any given period only with probability  $1 - \omega$ . Firms set their prices independent of other firms and of the time elapsed since the last adjustment. By considering the optimal price chosen by each firm, it is well-known, as shown by Galí (2002) and others, that this standard production sector specification leads to the following inflation adjustment equation, mostly known as the new Keynesian Phillips curve (NKPC):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\psi}_t, \quad (4)$$



where  $\pi_t$  is the rate of inflation between time  $t-1$  and  $t$ ,  $\hat{\psi}_t$  is the percentage deviation of real marginal cost around its steady state<sup>5</sup> (which is the same for all the firms) and  $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$ . The cost channel model deviates from the standard new Keynesian model in the specification of the marginal cost. The marginal cost is different in the cost channel model than in the standard model due to the borrowing of the wage bill. What follows is the derivation of the corresponding real marginal cost with regard to the cost channel model.

Assume a firm takes out a loan worth  $JW_tN_t$  from financial intermediaries to cover part of its nominal wages of  $W_tN_t$ . Accordingly,  $J$  ( $J \in [0, 1]$ ) denotes the portion of the wage bill covered by firms using external loans at time  $t$ . If  $J = 1$ , firms borrow the full wage bill externally. If  $J = 0$ , that means the firm does not take out loans externally to cover the wage bill.

Accordingly, the real marginal cost is given by:

$$\psi_t = \left(\frac{W_t}{P_t}\right) \left(\frac{N_t}{Y_t}\right) [1 + J(R_t - 1)] \quad (5)$$

The log linearised real marginal cost (see Appendix A for derivation) is:

$$\hat{\psi} = (\sigma + \eta)x_t + J\hat{R}, \quad (6)$$

where  $x_t$  is the output gap given by  $(\hat{Y}_t - \hat{Y}_t^f)$ . The percentage deviation of output around its steady state is  $\hat{Y}_t$ , and  $\hat{Y}_t^f$  is the percentage deviation of flexible price output around its steady state at time  $t$ .<sup>6</sup> The percentage point deviation of nominal interest rate around zero inflation steady state value of  $R$  is  $\hat{R}_t$ .

Accordingly, the NKPC adjusted for the cost channel, is derived using equations (4) and (6) as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\sigma + \eta)x_t + \kappa J \hat{R}_t, \quad (7)$$

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<sup>5</sup> Throughout this paper, a hat sign (^) denotes the percentage deviation of the concerned variable around its steady state.

<sup>6</sup> Equilibrium flexible price output is discussed in detail below.

It is clear from equation (7), that when  $J = 1$ , the NKPC boils down to Ravenna and Walsh (2006) and, when  $J = 0$ , it turns to the standard NKPC. Iterating this equation forward yields the following:

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E^t [(\sigma + \eta)x_{t+k} + J\hat{R}_{t+k}].$$

This equation shows that current inflation not only depends on the current and future path of the output gap but also on the current and future path of the nominal interest rates. The latter influences current inflation directly due to the inclusion of the cost channel in the model.

Log linearising the Euler equation given by (1) yields the well-known dynamic IS equation:

$$x_t = E_t x_{t+1} - \sigma^{-1}(\hat{R}_t - E_t \pi_{t+1}) + u_t,$$

where  $u_t$  is an exogenous demand disturbance term.

Since I am comparing the results with JTW, to be compatible with their model, I introduce natural rate of interest ( $r^n$ ) as defined by JTW.<sup>7</sup> Accordingly, the dynamic IS equation becomes this:

$$x_t = E_t x_{t+1} - \sigma^{-1}[\hat{R}_t - E_t \pi_{t+1} - \hat{r}_t^n], \quad (8)$$

where  $\hat{r}_t^n$  is the percentage point deviation of the net natural interest rate around its zero inflation steady state value of  $r^n$ . At the zero inflation steady state, nominal interest rate is equal to natural rate of interest rate.<sup>8</sup> Accordingly, at the zero inflation steady state, the following result holds:

$$R = 1 + r^n = \frac{1}{\beta}$$

### Aggregate Resource Constraint

The economy I consider in this model is a simple economy. It abstracts from aggregate demand components such as investments, government purchases or net exports. Accordingly, aggregate resource constraint of the economy is given by this:

<sup>7</sup> JTW defines the natural interest rate as follows:  $r_t^n = \sigma E_t [(y_{t+1}^p - y_t^p) - (g_{t+1} - g_t) + (\frac{1}{\beta} - 1)]$ , where  $y_t^p$  is the potential output and  $g_t$  is a disturbance that fluctuates independently of changes in the real interest rate.

<sup>8</sup> At the zero inflation steady state, I assume the potential growth in the economy to be zero and that there will be no disturbances to the natural rate of interest. Accordingly, the natural interest rate at zero in addition steady state is equal to  $\frac{1}{\beta} - 1$ . From the Euler equation given by equation (1), it is easy to and the zero in addition steady state value of the net nominal interest rate is also equal to  $\frac{1}{\beta} - 1$ .

$$Y_t = C_t$$

where  $Y_t$  is the aggregate production.

### Flexible Price Equilibrium

The model developed above is characterised by three distortions. The first of them is the presence of market power in the goods market due to the monopolistic competition among firms. The second is due to price rigidity. These two distortions are basic in the standard new Keynesian model. The third distortion is specific to this study, and it is due to the cost channel. In the following section, I relax the price rigidity assumption and examine the equilibrium output under flexible prices.

Suppose that all firms adjust prices optimally in each period, i.e. prices are fully flexible. When prices are fully flexible, all firms charge the same price. Each firm sets its price equal to a markup,  $\delta \left( = \frac{\theta}{\theta-1} > 1 \right)$  over its nominal marginal cost, which is constant over time. Hence, it follows that the real marginal cost will also be constant and equal to the inverse of the optimal markup chosen by firms.<sup>9</sup> Let superscript  $f$  denote the flexible price equilibrium values of relevant variables. Accordingly:

$$\frac{\left[ 1 + j(R^f - 1) \left[ \frac{w_t}{P_t} \right]^f \right]}{A_t} = \frac{1}{\delta}$$

Hence:

$$\left[ \frac{w_t}{P_t} \right]^f = \frac{A_t}{\delta [1 + j(1 + R_t^f - 1)]} \quad (9)$$

Households equate the real wage to the marginal rate of substitution between leisure and consumption. From equation (2):

$$\left[ \frac{w_t}{P_t} \right]^f = \frac{\chi N_t^\eta}{C_t^{-\sigma}} \quad (10)$$

Combining Equation (9) and (10) together with production function and resource constraint yields the following:

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<sup>9</sup> See Walsh (2010, pp. 334-335) for a detailed description of the flexible price mechanism.

$$\frac{A_t}{\delta[1 + J(1 + R_t^f - 1)]} = \frac{\chi \left[ \frac{Y_t^f}{A_t} \right]^\eta}{[Y_t^f]^{-\sigma}}$$

Hence:

$$Y_t^f = \left[ \frac{A_t^{(1+\eta)}}{\chi\delta[1 + J(1 + R_t^f - 1)]} \right]^{\frac{1}{\sigma+\eta}}$$

This shows that the equilibrium flexible price output is distorted by monetary policy as the nominal interest rate is an argument in the equation and the presence of market power in the goods market. With regard to the distortions by monetary policy, for example, an increase in nominal interest rate decreases labour demand, which in turn reduces the equilibrium level of flexible price output. This distortion is directly due to the inclusion of the cost channel in the model.

The steady state value of the flexible price output is given as follows:

$$Y^f = \left[ \frac{1}{\chi\delta[1 + J(R^f - 1)]} \right]^{\frac{1}{\sigma+\eta}}$$

where  $R^f$  is the steady state value of the flexible price nominal interest rate.

The steady state value of the flexible price output is also distorted by monetary policy and monopolistic competition. If  $J = 0$ , then by construction the cost channel is eliminated and the distortion is also eliminated. On the other hand, if the nominal interest rate is zero (or  $R^f = 1$ ), the distortion brought in by the cost channel is eliminated. Distortion due to monopolistic competition can be eliminated by setting  $\delta = 1$ .

Next, I specify the objective of the monetary authority and its problem.

### 3.3 Monetary Authority

The monetary authority has one monetary instrument, which is the short-term nominal interest rate. It attempts to minimise the loss function:

$$L_0 = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \{ \pi_t^2 + \lambda x_t^2 \} \quad (11)$$

where  $\lambda$  is a positive parameter representing the weight assigned to output stability. This loss function has been derived using second-order Taylor expansion of the utility of the

representative household. Woodford (2003) derived this for a standard new Keynesian model, while Ravenna and Walsh (2006) derived this for a new Keynesian model with a cost channel, similar to the present model.

### 3.4 Optimisation Problem

The central bank minimises equation (11) subject to equations (7), (8) and the ZLB condition. The problem is as follows:

$$\text{Min} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \{\pi_t^2 + \lambda x_t^2\}, \quad (12)$$

Subject to,

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma^{-1} (\hat{R}_t - E_t \pi_{t+1} - \hat{r}_t^n) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa (\sigma + \eta) x_t + \kappa J \hat{R}_t \end{aligned}$$

and

$$\hat{R}_t + R - 1 \geq 0. \quad (13)$$

This problem cannot be solved by applying standard solution methods for rational expectations models because of the complications brought in by the non-linear constraint in equation (13). To make the analysis more tractable, I consider the agents with perfect foresight under both discretion and commitment policies in the following sections.

In this analysis, following JTW, it has been considered that the economy is in a liquidity trap following a large negative demand shock to the natural interest rate. The natural rate of interest follows an AR(1) process following the shock and converges to steady state value in and after period one. The AR(1) process is as follows:

$$\hat{r}_t^n = \rho^t \epsilon_0 + r^n \text{ for } t = 0, 1, 2, 3, \dots, \quad (14)$$

where  $\epsilon_0$  is the large negative shock that occurs in the time  $t = 0$ , and  $\rho$  is the persistence of the shock ( $0 < \rho < 1$ ).

The optimisation problem under each commitment and discretionary policies is considered in the following sections.

#### Optimisation under Discretion

Under discretion, the central bank treats the optimisation problem as a sequential optimisation problem. Accordingly, the central bank makes whatever decision that is optimal in each period without committing to future actions. The central bank chooses

$(x_t, \pi_t)$  in order to minimise the objective function given by equation (12) subject to adjusted NKPC, the dynamic IS curve and the ZLB constraint. The Lagrangian method is used to solve this constrained optimisation problem. Accordingly, the problem with perfect foresight is as follows:

$$L = \beta^t \left\{ \frac{1}{2} (\pi_t^2 + \lambda x_t^2) + \mu_t [x_t - x_{t+1} + \sigma^{-1} (\hat{R}_t - \pi_{t+1} - \hat{r}_t^n)] + \delta_t [\pi_t - \beta \pi_{t+1} - \kappa(\sigma + \eta)x_t - \kappa J \hat{R}_t] + v_t (\hat{R}_t + R - 1) \right\}, \quad (15)$$

where  $\mu_t$ ,  $\delta_t$  and  $v_t$  are Lagrangian multipliers.

Under discretion, the central bank optimises in each period. Accordingly, the Karush-Kuhn-Tucker (KKT) conditions of the problem are the following:

$$\begin{aligned} \pi_t + \delta_t &= 0, \\ \lambda x_t + \mu_t - \kappa(\sigma + \eta)\delta_t &= 0, \\ \mu_t \sigma^{-1} - \kappa J \delta_t + v_t &= 0, \\ x_t - x_{t+1} + \sigma^{-1} (\hat{R}_t - \pi_{t+1} - \hat{r}_t^n) &= 0, \\ \pi_t - \beta \pi_{t+1} - \kappa(\sigma + \eta)x_t - \kappa J \hat{R}_t &= 0, \\ v_t (\hat{R}_t + R - 1) &= 0, \\ v_t &\leq 0, \\ \hat{R}_t + R - 1 &\geq 0. \end{aligned}$$

### Steady State under Discretion

At steady state, define  $x_t = x$ ,  $\pi_t = \pi$ ,  $r_t^n = r^n$ ,  $\delta_t = \delta$ ,  $\mu_t = \mu$ ,  $v_t = v$ ,  $\hat{R}_t = \hat{R}$  and  $\hat{r}_t^n = 0$ . Also define  $R_s$  as the value of the gross nominal interest rate relevant to the particular steady state.<sup>10</sup> Accordingly, the KKT conditions become the following:

$$\begin{aligned} \pi + \delta &= 0, \\ \lambda x + \mu - \kappa(\sigma + \eta)\delta &= 0, \\ \sigma^{-1}\mu - \kappa J \delta + v &= 0, \\ \pi &= \hat{R} \\ (1 - \beta)\pi - \kappa(\sigma + \eta)x - \kappa J \hat{R} &= 0. \end{aligned}$$

Potentially there can be two steady states in the system, an interior solution and a corner solution. First, I will consider the interior solution. In this case, the nominal interest

<sup>10</sup> According to the steady state definition of variables,  $R_{ss} = \hat{R} + R$ .

rate is strictly positive, i.e.  $R_{ss} > 1$ . According to the KKT conditions,  $\nu = 0$ . Substituting these into the above steady state conditions and solving the linear system of equations yields:

$$\pi = 0, x = 0, R_{ss} = 1 + r^n = R, \mu = 0, \delta = 0 \text{ and } \nu = 0.$$

In this steady state, inflation and output gap is zero. This steady state minimises the loss of the central bank's objective function.

Now turn to the corner solution. Here, the nominal interest rate has hit the zero lower bound, i.e.  $R_{ss} = 1$ . Accordingly,  $\hat{R} = 1 - R$ . The solution for  $\nu$  in the linear system of equations given above is (results for other variables are given in the appendix B):

$$\nu = - \frac{[\lambda(1 - \beta) + \kappa(J\lambda - \kappa(\eta + \sigma)(\eta + \sigma(1 - J)))]r^n}{\kappa\sigma(\eta + \sigma)}$$

As required by the KKT conditions,  $\nu$  is strictly negative at the corner solution for the following values of  $J$ :

$$J < \frac{\lambda(1 - \beta) + \kappa^2(\eta + \sigma)^2}{\kappa(\lambda + \kappa\sigma(\eta + \sigma))}$$

Therefore, there exists a second steady state at the ZLB when  $J$  is sufficiently small. JTW show that there is a second steady state under discretion for a no-cost channel economy, i.e. when  $J = 0$ . For the baseline parametrisation values set at the calibration section below, the maximum value of  $J$  to have a second steady state is 0.9. This steady state does not minimise the central bank loss function since both inflation and the output gap have been deviated from zero.

The Friedman rule [Friedman (1969)] of zero nominal interest rate is not optimal in this model. One reason for this different conclusion is the absence of any explicit role for money in the utility approximation of equation (11), as shown by Walsh (2010, p355). Another reason is, as mentioned by JTW, the central bank loss functions defined in these types of optimisation studies do not include the existence of shoe-leather cost. Friedman argues that distortions due to shoe-leather costs are proportional to nominal interest rates, and therefore, these distortions can be eliminated by setting nominal interest rate to zero.

### Optimisation under Commitment

Under commitment, the central bank optimises the system and commits to a current and future policy plan. I assume full credibility of the central bank. Accordingly, the central bank specifies the desired levels of inflation and the output gap for all possible dates and the states of nature. The central bank is assumed to choose a state contingent sequence  $\{x_t, \pi_t\}_{t=0}^{\infty}$  which minimises its objective function given by equation (12) subject to the adjusted NKPC, the dynamic IS curve and the ZLB constraint. Accordingly, the KKT conditions are as follows:

$$\begin{aligned} \pi_t - (\beta\sigma)^{-1}\mu_{t-1} + \delta_t - \delta_{t-1} &= 0, \\ \lambda x_t + \mu_t - \beta^{-1}\mu_{t-1} - \kappa(\sigma + \eta)\delta_t &= 0, \\ \sigma^{-1}\mu_t - \kappa f \delta_t + \nu_t &= 0, \\ x_t - x_{t+1} + \sigma^{-1}(\hat{R}_t - \pi_{t+1} - \hat{r}_t^n) &= 0, \\ \pi_t - \beta\pi_{t+1} - \kappa(\sigma + \eta)x_t - \kappa f \hat{R}_t &= 0, \\ \nu_t(\hat{R}_t + R - 1) &= 0, \\ \nu_t &\leq 0, \\ \hat{R}_t + R - 1 &\geq 0. \end{aligned}$$

Since lagged values of the Lagrange multipliers appear in the KKT conditions, it is clear that the KKT conditions are history dependent. Accordingly, the optimal choice of inflation, the output gap and the nominal interest rate depend on the past values of the endogenous variables. If the central bank deviates from its policy plan (a credibility loss), the outcome would be different.

### Steady State under Commitment

Consider the steady state variables defined under discretionary policy. In the steady state under commitment, the KKT conditions derived above become these:

$$\begin{aligned} \pi - (\beta\sigma)^{-1}\mu &= 0, \\ \lambda x + (1 - \beta^{-1})\mu - \kappa(\sigma + \eta)\delta &= 0, \\ \sigma^{-1}\mu - \kappa f \delta + \nu &= 0, \\ \pi &= \hat{R}, \\ (1 - \beta)\pi - \kappa(\sigma + \eta)x - \kappa f \hat{R} &= 0. \end{aligned}$$



It can be shown that the interior solution (i.e.  $R_{ss} > 1$ ) is exactly the same as in the discretionary case as follows:

$$\pi = 0, x = 0, R_{ss} = 1 + r^n = R, \mu = 0, \delta = 0 \text{ and } \nu = 0.$$

This shows that under each policy, the interior solution converges to the same steady state with zero inflation and output gap minimising the central bank loss function.

Now turn to the corner solution under commitment. The solution for  $\nu$  is as follows:

$$\nu = \frac{[J(1 + \kappa)\lambda + \kappa\sigma(\eta + \sigma) + \beta(J\lambda + \kappa(\eta + \sigma)(\eta + \sigma(1 - J)))]r^n}{\kappa(\eta + \sigma)^2}$$

The sign of  $\nu$ , which is the Lagrangian multiplier of the ZLB constraint, is strictly positive. This contradicts the KKT conditions. Therefore, under commitment there does not exist a second steady state at the ZLB.

### 3.5 Optimal Path under Discretion

Consider that a large negative shock occurred to the natural interest rate, which converges to its steady state level over time. Assume the ZLB is binding until time  $T^d$  (i.e.,  $t = 0, 1, \dots, T^d$ ) and not binding thereafter (i.e.,  $t \geq T^d + 1$ ).

Consider the dynamic path where  $t \geq T^d + 1$ . Here,  $\nu_t = 0$ . Accordingly, the KKT conditions under discretion can be stated as follows:

$$\pi_t + \delta_t = 0, \quad (16)$$

$$\lambda x_t + \mu_t - \kappa(\sigma + \eta)\delta_t = 0, \quad (17)$$

$$\mu_t \sigma^{-1} - \kappa J \delta_t = 0, \quad (18)$$

$$x_t - x_{t+1} + \sigma^{-1}(\hat{R}_t - \pi_{t+1} - \hat{r}_t^n) = 0, \quad (19)$$

$$\pi_t - \beta \pi_{t+1} - \kappa(\sigma + \eta)x_t - \kappa J \hat{R}_t = 0. \quad (20)$$

From equations (16)–(18):

$$\lambda x_t + \kappa[\sigma(1 - J) + \eta]\pi_t = 0. \quad (21)$$

Combining this result with equations (19) and (20) yields:

$$\pi_{t+1} = \tau \pi_t - \zeta \hat{r}_t^n, \quad (22)$$

$$\text{Where } \tau = \frac{J\kappa\sigma a + \lambda - \kappa(\sigma + \eta)a}{J\kappa\sigma a + J\lambda\kappa + \lambda\beta}, \zeta = \frac{J\lambda\kappa}{J\kappa\sigma a + J\lambda\kappa + \lambda\beta} \text{ and } a = -\kappa[\sigma(1 - J) + \eta].$$

When  $J = 0$ , i.e. when firms do not borrow externally to finance the wage bill, equation (22) is identical to JTW, which is given by  $\pi_{t+1} = \beta^{-1} \left( 1 + \frac{\kappa^2(\sigma+\eta)^2}{\lambda} \right) \pi_t$ . Since  $\tau$  is always greater than unity as shown by JTW, this difference equation has a bounded solution which is given by  $\pi_t = 0$ .

However, when  $J > 0$ ,  $\tau$  is not necessarily greater than unity. The value depends on the parametrisation. To have an idea about the value of  $\tau$ , I plot the value of the coefficient with different values of  $J$  given in Figure 1.<sup>11</sup> It shows that the value of  $\tau$  is greater than unity for smaller values of  $J$  (when  $J < 0.59$ ), while it is less than unity for higher values of  $J$ .

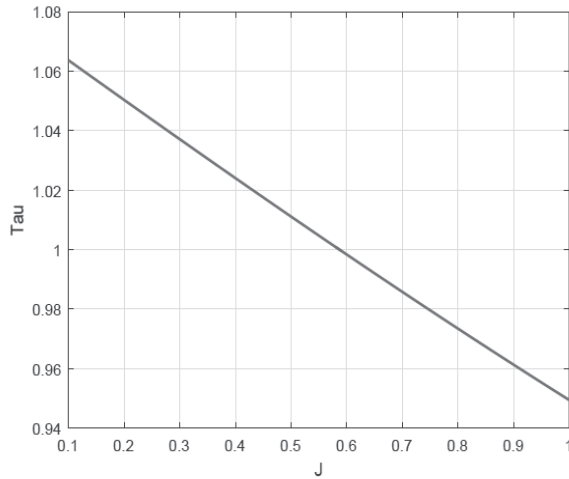
The fact that  $\tau$  becomes less than unity for higher values of  $J$  reveals an important difference between cost channel and no-cost channel economies. First, for larger values of  $J$ , equation (22) has a stationary solution. This means  $\pi_{T^d+1}$  does not necessarily take the value zero as shown in the case  $J = 0$ . Further, according to equation (21),  $x_{T^d+1}$  does not take the value zero. Accordingly, when  $J > 0$ , the central bank may not necessarily increase interest rates at time  $T^d+1$ . Therefore, when  $J$  is larger, the central bank does not necessarily increase interest rates one-to-one with the exogenous natural rate of the interest rate. Nakov (2008) found a similar result under discretionary policy when he considered that the natural interest rate follows a stochastic AR(1) process; however, he did not consider a cost channel economy.

Further, the fact that  $|\tau| < 1$  means that there are multiple equilibria under discretionary policy. This leads to the equilibrium policy path selection. In the following simulation exercise, I consider the policy in which the economy returns to the zero inflation steady state on or before the 100<sup>th</sup> quarter.

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<sup>11</sup> The parameters are set to baseline values as defined in section 4.1.

**Figure 1:  $J$  vs  $\tau$  for baseline parametrisation**



## 4. Simulation and Results

### 4.1 Calibration

I calibrate the model for the US economy, with the baseline specification given in Table 1. The parameter values are within the standard new Keynesian parameter values and are carried over from Ravenna and Walsh (2006) and JTW. Still, the choice of two parameter values is worth noting here. Firstly, the weight on output in the loss function, ( $\lambda$ ) is set at 0.25 in the baseline calibration. However, underlying theory implies a much smaller value for  $\lambda$ .<sup>12</sup> Most monetary policy literature, including Ravenna and Walsh (2006), employs a large value for  $\lambda$  considering the empirical relevance. Accordingly, I chose a large value for  $\lambda$ . Secondly, as mentioned before, the zero inflation steady state value of the natural interest rate is the value that has been calculated under the assumption that there is no growth in the potential output. Accordingly, the steady state value of the natural rate of interest is set at  $\frac{1}{\beta} - 1$ . These values are based on a time period equal to three months (one quarter).

<sup>12</sup> Theoretical value of  $\lambda = \frac{(1-\beta\omega)(1-\omega)(\sigma+\eta)}{\omega^\theta} = 0.01$ , when  $\theta = 11$ . See Ravenna and Walsh (2006) for the derivation of the theoretical value of  $\lambda$ .

**Table 1: Parametrisation**

Parameter	Description	Domain	Baseline Value
$\beta$	Discount rate in the utility function	(0,1)	0.99
$\sigma$	Coefficient of relative risk aversion	(0, $\infty$ )	1
$\eta$	Elasticity of labour supply in the utility function	(0, $\infty$ )	0.5
$\omega$	Share of firms that cannot adjust prices optimally	[0,1)	0.75
$\varkappa$	Slope parameter used in NKPC	(0, $\infty$ )	0.09
$\lambda$	Weight on output in the loss function	(0, $\infty$ )	0.25
$\varrho$	Natural interest rate shock persistence parameter	[0,1)	0.7
$r^n$	Steady state value of the natural rate of interest	[0, $\infty$ )	$\frac{1}{\beta} - 1$
$J$	Share of working capital to be financed externally	[0,1]	1

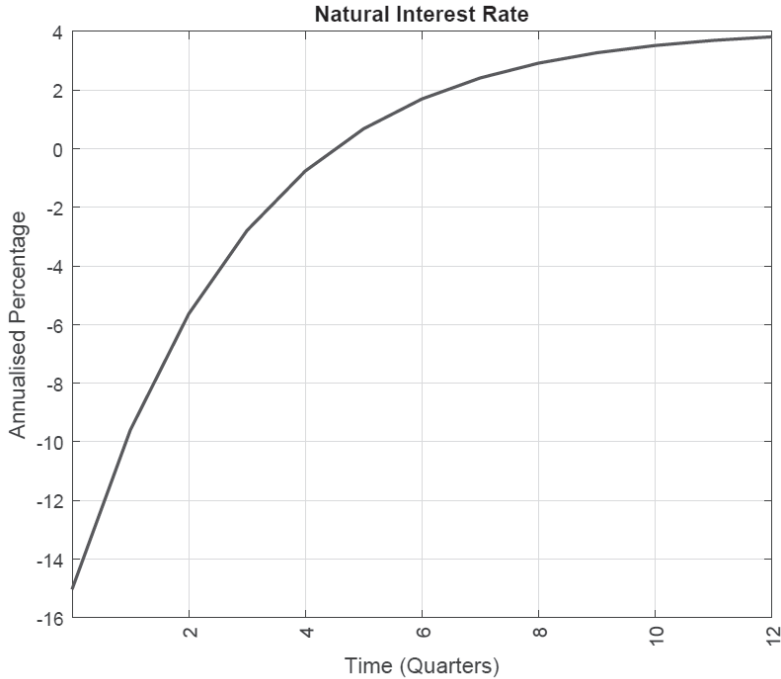
## 4.2 Simulation

In the baseline simulation, I consider the initial shock to the economy of the size of  $\epsilon_0 = -0.05$ , which is equivalent to around a 19 per cent drop in the annualised natural interest rate. In these simulations, I consider three values for  $J$ .<sup>13</sup> They are two extreme values  $J = 0$  and  $J = 1$ , and a more empirically relevant value of  $J = 0.6$ . The dynamic path of the exogenous natural interest rate due to the large negative demand shock is depicted in Figure 2.<sup>14</sup> The figure shows that the natural interest rate drops to -15 per cent and returns to a positive level by the fourth quarter following the shock.

<sup>13</sup> I used the software Matlab (version R2016a) to facilitate the simulations.

<sup>14</sup> Relevant values given in the results are in annualised figures.

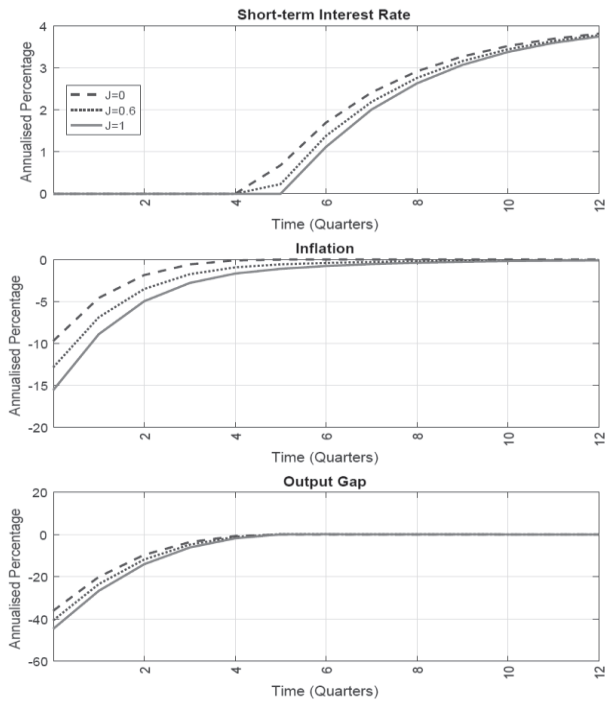
**Figure 2: Path of natural rate of interest**



### 4.3 Dynamic Path under Discretion

The dynamic path of the variables under discretionary policy is depicted in Figure 3. The solid lines depict the dynamics of the case when the cost channel of monetary policy is active (i.e.  $J = 1$ ), the dashed lines depict the case in which such channel is not active (i.e.  $J = 0$ ) while the dotted lines depict the case  $J = 0.6$ . The case  $J = 0$  is identical to JTW. Notice that, following the demand shock, the economy gets into a deeper recession in a cost channel economy. This is observed in inflation and output gap plots. The reason for that is, when the cost channel is active, a sudden drop in nominal interest rates directly affects the cost of production negatively. Consequently, it amplifies deflation. Accordingly, under discretion, the central bank has to keep short-term nominal interest rates at zero level longer in a cost channel economy to minimise the loss. This is the main difference of the policy between cost channel and no-cost channel economies under discretion. The top plot in the panel shows that the central bank exits the zero interest rates regime in the fifth quarter in a cost channel economy compared to the fourth quarter in a no-cost channel economy.

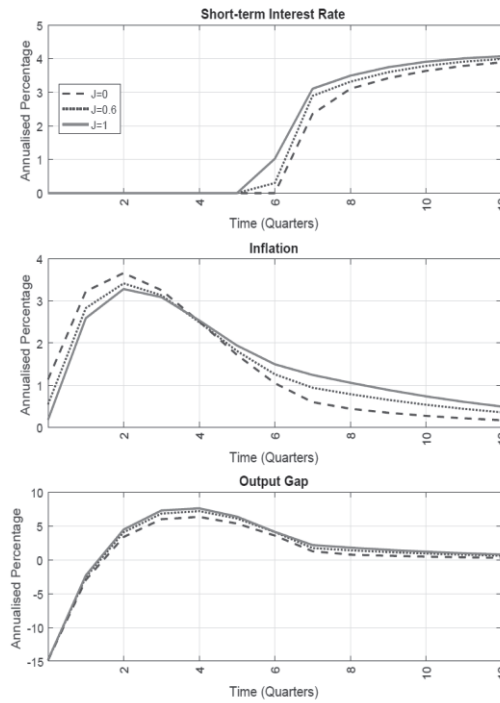
**Figure 3: Path of variables under discretionary policy**



#### 4.4 Dynamic Path under Commitment

Now I turn to the commitment policy. The reaction of the model economy due to a large negative shock under commitment is depicted in Figure 4. The main result is opposite to the discretionary regime. Now the central bank in a cost channel economy exits the zero nominal interest rate regime earlier than the no-cost channel economy. This is observed in the bottom plot, which shows the path of the nominal interest rate. The reason for that is, in a cost channel economy, agents expect higher inflation once the central bank starts exiting zero interest rates. Because under commitment policy agents expect more inflation in a cost channel economy, the central bank promises to terminate zero interest rate policy early. This fact is confirmed in the inflation plot under commitment. When the central bank starts increasing short-term interest rates, the cost channel economy experiences a higher inflationary regime. Accordingly, the top plot in the panel shows that the central bank exits the zero interest rates regime in the fifth quarter in a cost channel economy compared to the sixth quarter in a no-cost channel economy.

Figure 4: Path of variables under commitment policy



Under commitment, both in cost channel and no-cost channel economies, the monetary authority attempts to stabilise the output gap and inflation in the short-term. However, under discretion, the monetary authority attempts to stabilise inflation and the output gap in the medium-term. These facts are observed in Figure 3 and Figure 4. For example, under discretion, the initial drop in inflation is -15 per cent, compared to zero inflation under commitment in cost channel economies.

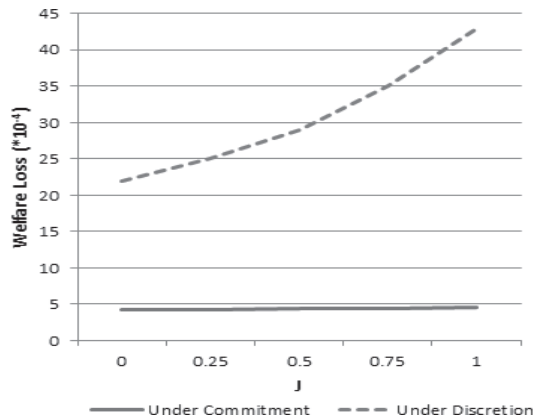
#### 4.5 Welfare Losses

In this section, the welfare losses under optimal policy are considered. I consider welfare losses in two ways. Firstly, the more natural measurement of welfare loss in these kind of models, i.e. by evaluating the central bank's objective function given in equation (11). However, the welfare units found in that way do not have a proper interpretation. Therefore, I also consider the consumption equivalent welfare loss.

Figure 5 depicts welfare losses of optimal monetary policy at the ZLB under both discretion and commitment for alternative values of  $J$  evaluated using the central bank loss function. The figure shows a well-known result in the optimal monetary policy literature at the ZLB: that welfare loss under commitment policy is less than under discretion. The reason for this is that the use of expected inflation is unavailable under the discretionary policy, as there is no incentive to implement promised inflation ex-post. The ZLB, therefore, generates significant additional welfare losses under discretionary policy.

With regard to the cost channel, welfare loss under both discretion and commitment is high compared to no-cost channel economies (compare the cases when  $J = 0$  and  $J = 1$  in Figure 5). The negative impact of the cost channel on welfare under discretionary policy is substantially high compared to its impact under commitment policy. In cost channel economies, under commitment welfare loss increases by only 9 per cent, compared to a 95 per cent increase under discretion. Demirel (2013) in a different context also found that, in a cost channel economy, a switch from discretion to commitment in monetary policy yields greater welfare gains relative to a no-cost channel economy.

**Figure 5: Welfare loss for different values of  $J$**



In the consumption equivalent of welfare analysis, I consider the percentage loss of consumption under each policy compared to the steady state consumption.<sup>15</sup> In line with

<sup>15</sup> Method of calculation of consumption equivalent welfare is based on Adam and Billi (2007). Adam and Billi (2007, Page 748) show that the utility equivalent percentage loss of consumption in the steady state is given by:  $p = 100 * \frac{1}{\sigma} \left( -1 + \sqrt{1 + \frac{2(1-\beta)L'}{1/\sigma}} \right)$ , where  $L' = \frac{1}{2} \frac{\omega\theta(1+\zeta\theta)}{(1-\omega)(1-\omega\beta)} \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2)$  and  $\zeta$  is elasticity of a firms' real marginal cost. In addition to the baseline parameterisation given in Table 11, following Adam and Billi (2007), I set  $\theta = 7.66$  and  $\zeta = 0.47$  for this calculation.



above results, I find under commitment policy in a cost channel economy, the loss of consumption is 0.037 percent compared to 0.033 percent loss without the cost channel. Under discretion, with the cost channel, loss of consumption is 0.428 percent compared to 0.244 percent loss without cost channel.

#### 4.6 Sensitivity Analysis

In order to determine the robustness of the results, I perform some sensitivity analysis in this section. I examine various values of parameters and consider the exit timing of a zero interest rate regime in cost channel and no-cost channel economies. I start with alternative sizes of shocks and their persistence. Table 2 presents results.<sup>16</sup> As expected, the table shows that, when the shock size is high and persistent, it takes longer to exit the zero interest rate regime. Further, the result that I found for the baseline parametrisation is valid here too. That is, under discretionary policy, it takes an equal or longer time to exit a zero interest rates regime in a cost channel economy compared to a no-cost channel economy. In contrast, under commitment policy, it takes a shorter or equal time to exit a zero interest rate regime in a cost channel economy.

**Table 2: Sensitivity Analysis: Shock Size and its Persistence**

		Shock and its Persistence											
		$\epsilon = -0.02$			$-0.05$			$-0.1$			$-0.3$		
		$\rho = 0$	0.5	0.7	$\rho = 0$	0.5	0.7	$\rho = 0$	0.5	0.7	$\rho = 0$	0.5	0.7
$T^d$	J=0	0	0	1	0	2	4	0	3	6	0	4	9
	J=1	0	1	2	0	2	5	0	3	7	0	5	10
$T^c$	J=0	0	1	2	1	3	6	2	5	8	4	7	11
	J=1	0	1	2	1	3	5	2	4	7	3	6	10

Next, I examine the sensitivity of results to the following variables: a share of firms who cannot optimise prices in each period and labour supply elasticity under commitment policy. The results are given in Table 3. The table shows that when the prices are relatively flexible (when  $\omega$  takes relatively smaller values) and also when they are relatively rigid (when  $\omega$  takes relatively larger values), there is no difference between a cost channel economy and a no-cost channel economy.

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<sup>16</sup> Recall  $T^d(T^c)$  is the time, which denotes that the ZLB is binding under discretionary (commitment) policy.

**Table 3: Sensitivity Analysis: Price Rigidity and Labour Supply Elasticity under Commitment**

		Price Rigidity		
		$\omega=0.3$	$\omega=0.75$	$\omega=0.9$
$\eta=0.01$	J=0	5	6	6
	J=1	5	5	6
$\eta=0.5$	J=0	5	6	6
	J=1	5	5	6
$\eta=1$	J=0	5	6	6
	J=1	5	5	6

I also considered the sensitivity of results with regard to the discount factor. Results do not change to various values of  $\beta$ , a result that was found by JTW for a no-cost channel economy. It is confirmed here for the cost channel economies too.

#### 4. Conclusion

In this study, I incorporated the cost channel of monetary policy into an otherwise standard new Keynesian model and evaluated the optimal monetary policy at the zero lower bound on nominal interest rates. The novelty of the study is that this is the first time a new Keynesian type study has been performed to analyse the optimal monetary policy at the ZLB with the cost channel. I considered that the economy was initially in a recession with a liquidity trap following a large negative demand shock. The solution methodology was different to the standard new Keynesian model as the ZLB brings non-linearity into the model. I followed the JTW solution methodology in a perfect foresight environment, which solves the problem considering that the economy is already at the ZLB.

The study reveals some important results in the conduct of the optimal monetary policy in a cost channel economy at the ZLB. Importantly, the discretionary policy requires central banks to keep interest rates at the zero lower bound for longer in a cost channel economy. This is because, in cost channel economies, the deflation is high and persistent due to a large negative demand shock compared to no-cost channel economies. Further, cost channel economies introduce a policy trade-off between inflation and output gap. This result contradicts the finding by JTW that short-term interest rates follow a one-to-one exogenous natural rate of interest following a negative demand shock in a no-cost channel economy.

Under the commitment policy with a fully credible monetary authority, the simulation exercise has shown that the central bank is able to terminate the zero interest rate regime earlier in a cost channel economy than otherwise. This result is in contrast to the results

found under discretionary policy. The reason for that is, in a cost channel economy, the private sector has inflated inflationary expectations. This is because the cost channel increases future cost of production, and in turn, inflation when the central bank starts tightening monetary policy on a future known date.

Welfare losses are calculated using the central bank's objective function. It was revealed that the cost channel generates substantially high welfare losses, both under discretionary and commitment policies. Accordingly, abstracting the cost channel in these types of models can lead to under-estimation of welfare losses.

The robustness of the results was examined using a sensitivity analysis. The basic results found in the baseline parametrisation were confirmed in the sensitivity analysis. It was found that the difference between a cost channel economy and a no-cost channel economy is marginal with regard to the timing of the termination of zero interest rates when prices are relatively flexible or relatively rigid.

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## Appendices

### A.1 Log linearising real marginal cost

Taking log of (5) and substituting  $A_t = \frac{Y_t}{N_t}$  yield:

$$\ln\psi_t = \ln\left[\frac{W_t Y_t}{P_t N_t}\right] + \ln[1 + J(R_t - 1)] \quad (23)$$

For simplification purposes denote (23) as follows:

$$\ln\psi_t = \ln S_t + \ln V_t, \quad (24)$$

where  $S_t = \left[\frac{W_t Y_t}{P_t N_t}\right]$  and  $V_t = [1 + J(R_t - 1)]$ .

At steady state (24):

$$\ln\psi = \ln S + \ln V. \quad (25)$$

Log linearised equation given by taking the difference of (24) and (25):

$$\hat{\psi}_t = \hat{s}_t + \hat{v}_t. \quad (26)$$

Now consider  $\hat{s}_t$ :

$$\hat{s}_t = \hat{w}_t - \hat{p}_t + \hat{y}_t - \hat{n}_t,$$

Using (2),  $\hat{s}_t = \eta\hat{n}_t + \sigma\hat{w}_t$ . Define  $x_t$  as output gap leads to:

$$\hat{s}_t = (\sigma + \eta)x_t.$$

Now consider  $\hat{v}_t$ :

$$\hat{v}_t = \ln[1 + J(R_t - 1)] - \ln[1 + J(R - 1)],$$

$$\hat{v}_t \approx J(R_t - 1) - J(R - 1),$$

$$\hat{v}_t \approx J(R_t - R),$$

$$\hat{v}_t \approx J\hat{R}_t.$$

Substituting this result in (26) yields:

$$\hat{\psi}_t = (\sigma + \eta)x_t + J\hat{R}_t$$

**A.2 Results for corner solution under discretion**

$$\pi = -r^n,$$

$$x = -\frac{(1 - \beta + J\kappa)r^n}{\kappa(\eta + \sigma)},$$

$$R = 1,$$

$$\mu = -\beta\sigma r^n,$$

$$\delta = -\frac{r^n(-1 + \beta + J\kappa)\lambda + \kappa\sigma(1 - \beta)(\eta + \sigma)}{\kappa^2(\eta + \sigma)^2}.$$